# First fractions - half, halves
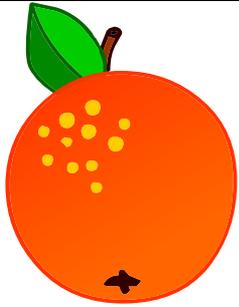
The orange has been cut in half.
A half is one of two equal parts.
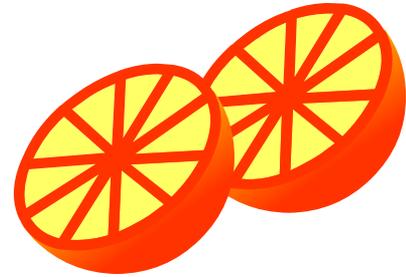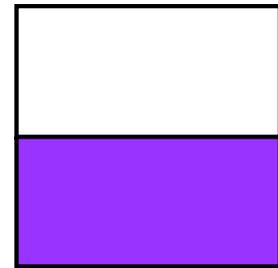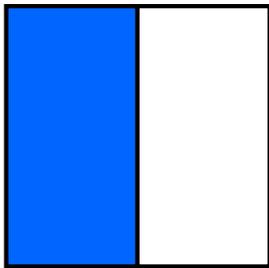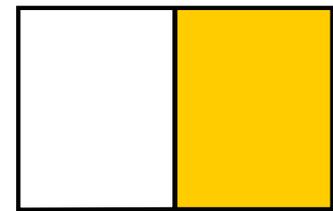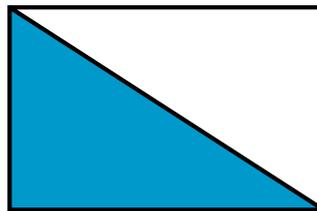
## Examples

$$\frac{1}{2}$$
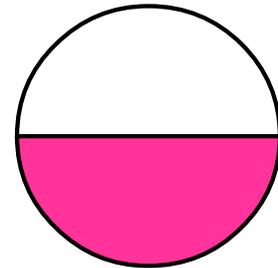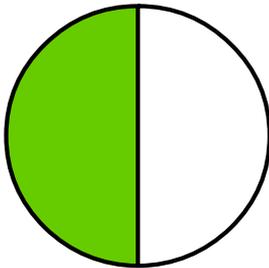
2 halves = 1 whole
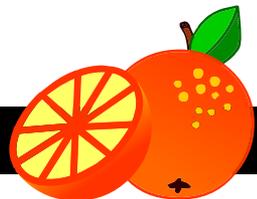
# First fractions - quarter, quarters
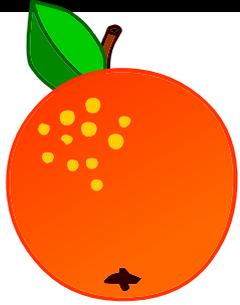
**The orange has been cut in quarters.
A quarter is one of four equal parts.**
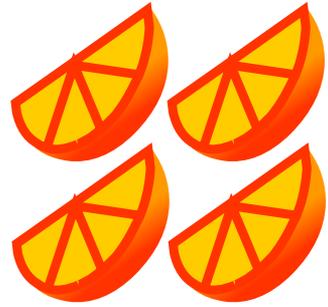
## Examples
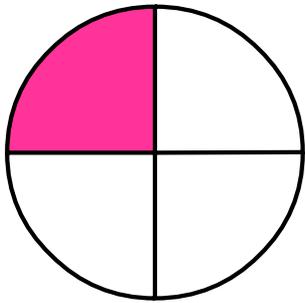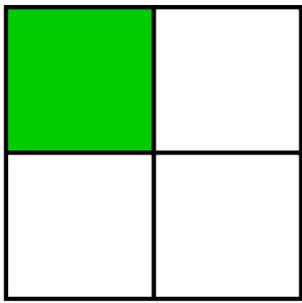
$$\frac{1}{4}$$
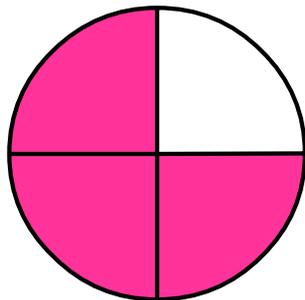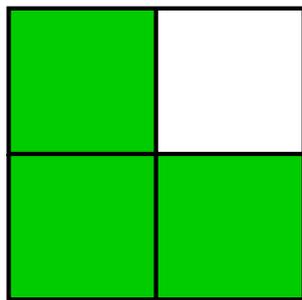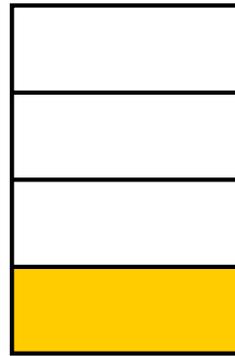
$$\frac{3}{4}$$
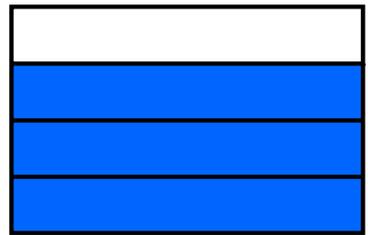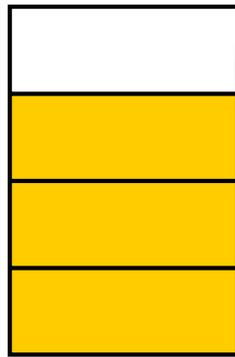
**4 quarters = 1 whole**
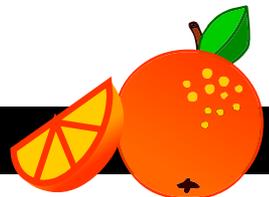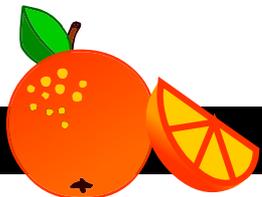
# First fractions - third, thirds
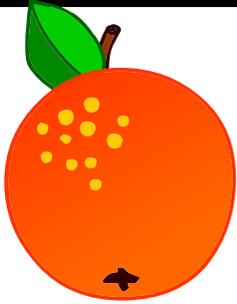
The orange has been cut in thirds.
A third is one of three equal parts.
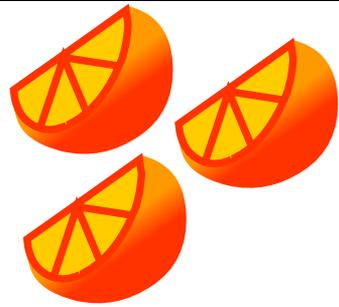
## Examples
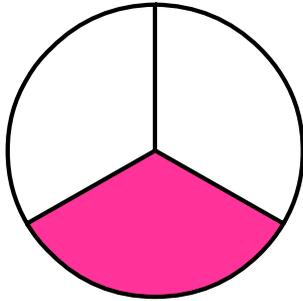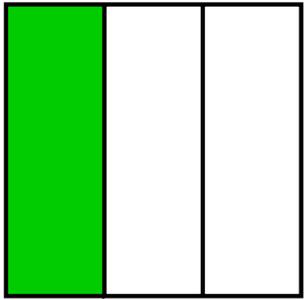
$$\frac{1}{3}$$
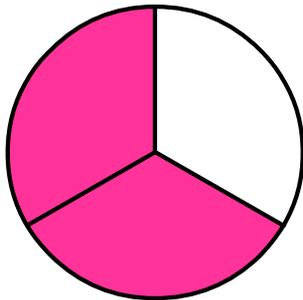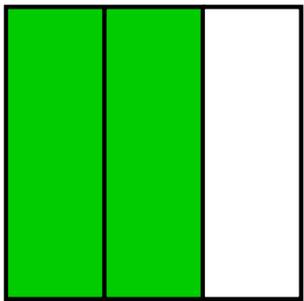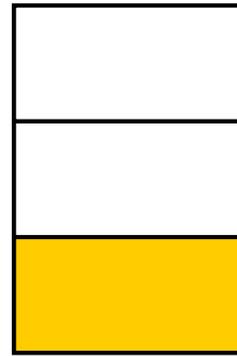
$$\frac{2}{3}$$
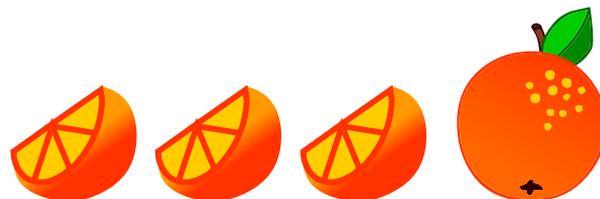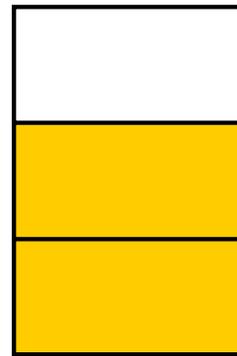
3 thirds = 1 whole
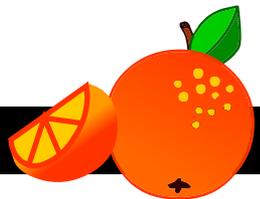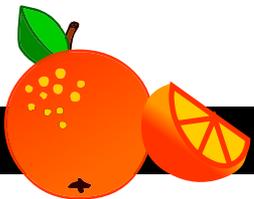
# Fractions - definition and basic terms
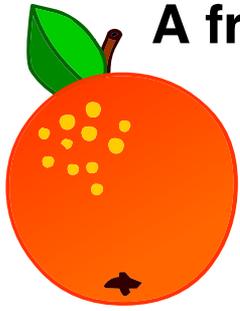
## A fraction is any part of a group, number or whole.

One orange has been cut in half.
A half is a fraction.
We write one half as

**The top number is called the numerator.**
$$\frac{1}{2}$$
**It is the number of parts we have.**

**The bottom number is called the denominator.**
**It is the total number of parts the whole is divided into.**

## More fractions
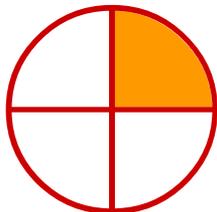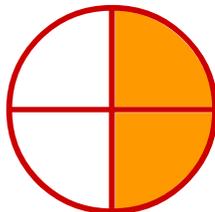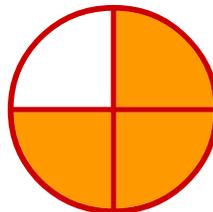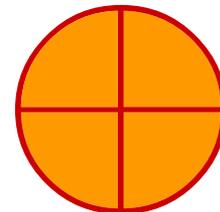
$\frac{1}{4}$  $\frac{2}{4}$  $\frac{3}{4}$  $\frac{4}{4}$
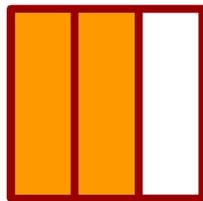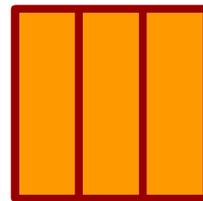
$\frac{1}{3}$  $\frac{2}{3}$  $\frac{3}{3}$
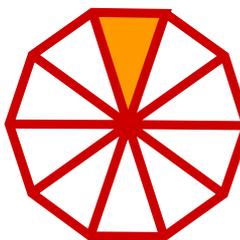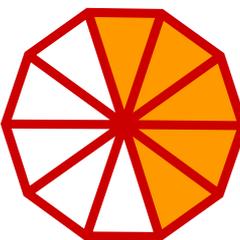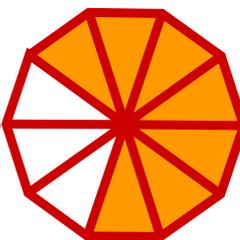
$\frac{1}{10}$  $\frac{3}{10}$  $\frac{5}{10}$  $\frac{7}{10}$  $\frac{10}{10}$
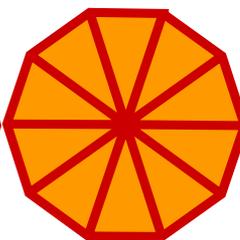
# Types of fractions

**There are three main types of fractions.**
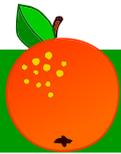
## Proper fraction

$\dfrac{1}{2}$   numerator   denominator

**The numerator is less than the denominator.**
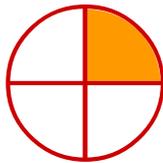
$\dfrac{1}{4}$     $\dfrac{2}{3}$     $\dfrac{7}{10}$

## Improper fraction

$\dfrac{5}{2}$   numerator   denominator

**The numerator is larger than or equal to the denominator.**
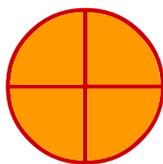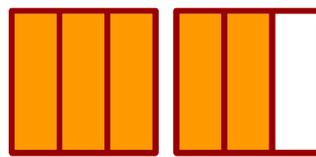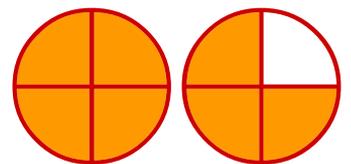
$\dfrac{4}{4}$     $\dfrac{5}{3}$     $\dfrac{7}{4}$
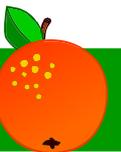
## Mixed number

$2\dfrac{1}{2}$

**A number written as a whole number with a proper fraction.**
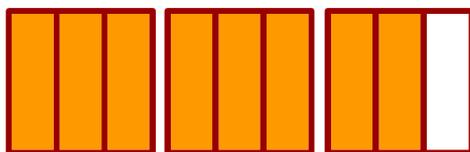
$2\dfrac{2}{3}$     $2\dfrac{3}{4}$
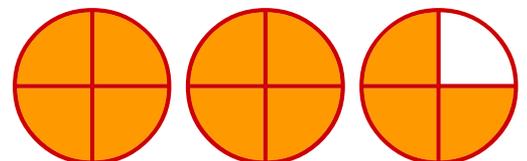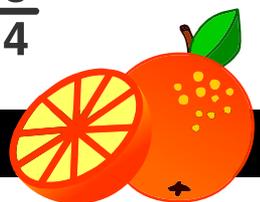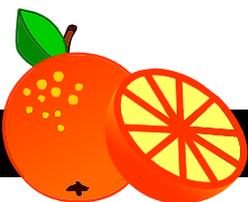
# Conversions

## Mixed number to improper fraction

1. **Multiply the whole number by the denominator.**
2. **Add to the numerator.**

$$2\frac{1}{2}$$

whole number

numerator

denominator

**Mixed number**
**A whole number with a proper fraction.**

$$\frac{5}{2}$$

numerator

denominator

**Improper fraction**
**Numerator is larger than or equal to the denominator.**

## Improper fraction to mixed number.

1. **Divide the numerator by the denominator.**
2. **Write as a whole number with fraction remainder.**

$$\div \frac{5}{2}$$

numerator

denominator

**Improper fraction**
**Numerator is larger than or equal to the denominator.**

$$2\frac{1}{2}$$

whole number

numerator

denominator

**Mixed number**
**A whole number with a proper fraction.**

# Equivalent Fractions

## Equivalent fractions have the same value.

| 1 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{1}{2}$ | | | | | | $\frac{1}{2}$ | | | | | |
| $\frac{1}{4}$ | | | $\frac{1}{4}$ | | | $\frac{1}{4}$ | | | $\frac{1}{4}$ | | |
| $\frac{1}{8}$ | | $\frac{1}{8}$ | | $\frac{1}{8}$ | | $\frac{1}{8}$ | | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| $\frac{1}{3}$ | | | | $\frac{1}{3}$ | | | | $\frac{1}{3}$ | | | |
| $\frac{1}{6}$ | | $\frac{1}{6}$ | | $\frac{1}{6}$ | | $\frac{1}{6}$ | | $\frac{1}{6}$ | | $\frac{1}{6}$ | |
| $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ |
| $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | | |
| $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | | |

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \frac{16}{32}$$
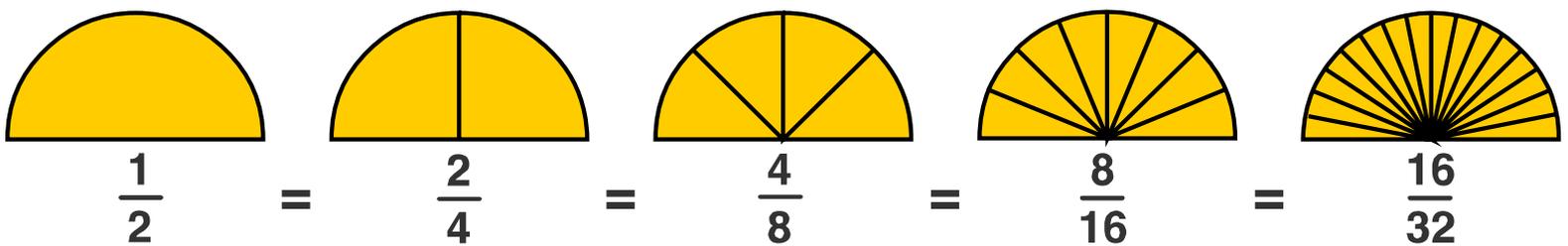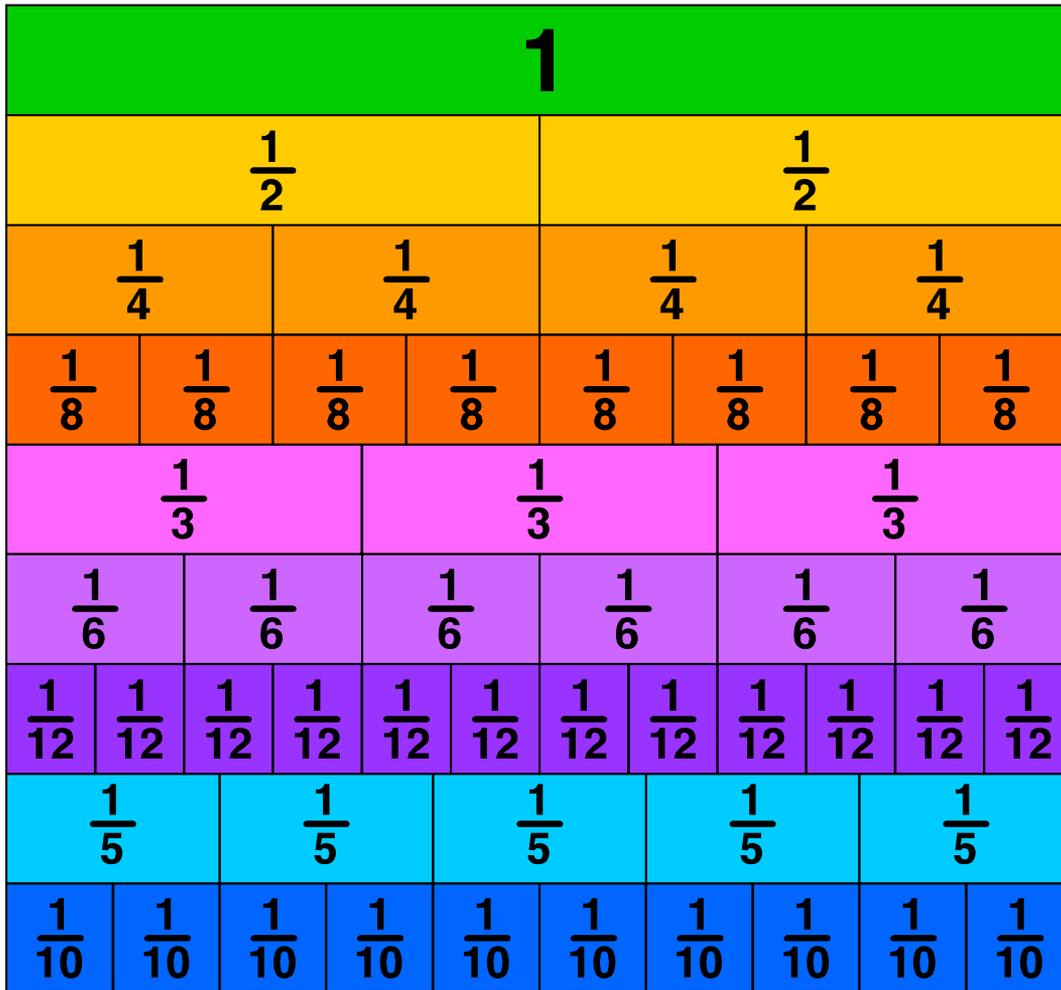
## You can make equivalent fractions by multiplying the numerator and the denominator by the same number.

$$\frac{1}{3} \xrightarrow{\times 2} \frac{2}{6} \xrightarrow{\times 2} \frac{4}{12} \xrightarrow{\times 2} \frac{8}{24} \xrightarrow{\times 2} \frac{16}{48}$$
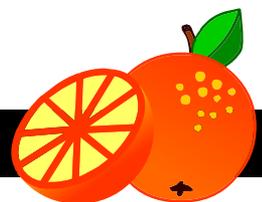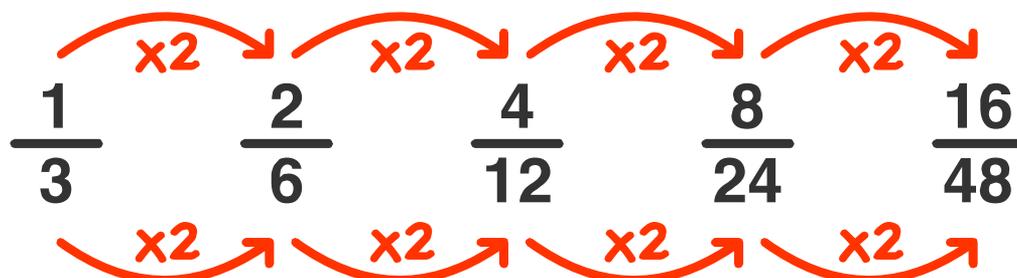
# Ordering proper fractions

**Proper fractions can be ordered on a number line between 0 and 1.**
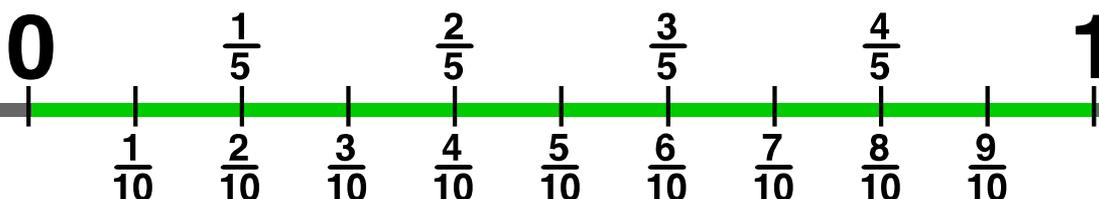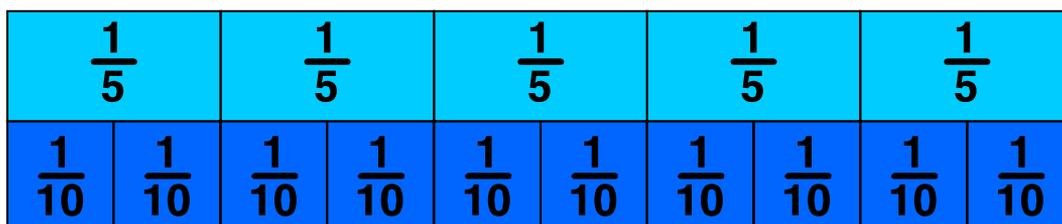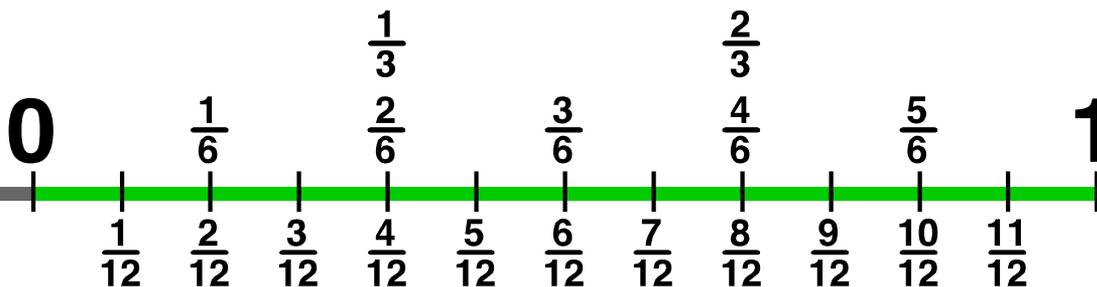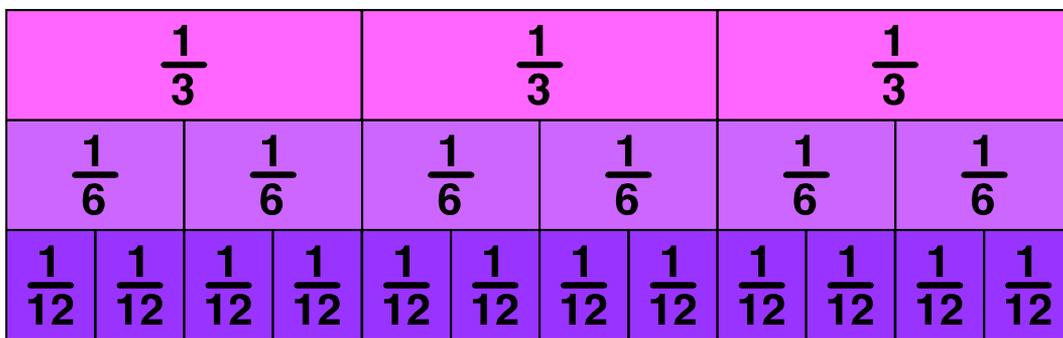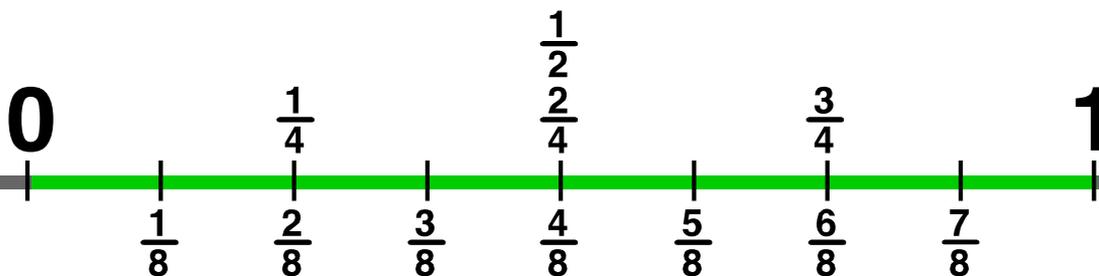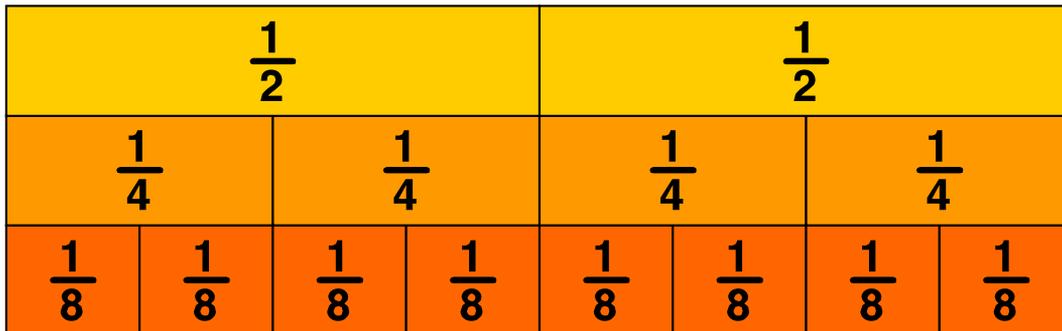
| $\frac{1}{2}$ | | $\frac{1}{2}$ | |
|---|---|---|---|
| $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| $\frac{1}{8}$ $\frac{1}{8}$ | $\frac{1}{8}$ $\frac{1}{8}$ | $\frac{1}{8}$ $\frac{1}{8}$ | $\frac{1}{8}$ $\frac{1}{8}$ |

0   $\frac{1}{4}$   $\frac{1}{2}$ $\frac{2}{4}$ $\frac{4}{8}$   $\frac{3}{4}$   1

$\frac{1}{8}$   $\frac{2}{8}$   $\frac{3}{8}$   $\frac{5}{8}$   $\frac{6}{8}$   $\frac{7}{8}$

| $\frac{1}{3}$ | | $\frac{1}{3}$ | | $\frac{1}{3}$ | |
|---|---|---|---|---|---|
| $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |
| $\frac{1}{12}$ $\frac{1}{12}$ | $\frac{1}{12}$ $\frac{1}{12}$ | $\frac{1}{12}$ $\frac{1}{12}$ | $\frac{1}{12}$ $\frac{1}{12}$ | $\frac{1}{12}$ $\frac{1}{12}$ | $\frac{1}{12}$ $\frac{1}{12}$ |

0   $\frac{1}{6}$   $\frac{2}{6}$ $\frac{1}{3}$   $\frac{3}{6}$   $\frac{4}{6}$ $\frac{2}{3}$   $\frac{5}{6}$   1

$\frac{1}{12}$ $\frac{2}{12}$ $\frac{3}{12}$ $\frac{4}{12}$ $\frac{5}{12}$ $\frac{6}{12}$ $\frac{7}{12}$ $\frac{8}{12}$ $\frac{9}{12}$ $\frac{10}{12}$ $\frac{11}{12}$

| $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ |
|---|---|---|---|---|
| $\frac{1}{10}$ $\frac{1}{10}$ | $\frac{1}{10}$ $\frac{1}{10}$ | $\frac{1}{10}$ $\frac{1}{10}$ | $\frac{1}{10}$ $\frac{1}{10}$ | $\frac{1}{10}$ $\frac{1}{10}$ |

0   $\frac{1}{5}$   $\frac{2}{5}$   $\frac{3}{5}$   $\frac{4}{5}$   1

$\frac{1}{10}$ $\frac{2}{10}$ $\frac{3}{10}$ $\frac{4}{10}$ $\frac{5}{10}$ $\frac{6}{10}$ $\frac{7}{10}$ $\frac{8}{10}$ $\frac{9}{10}$

# Simplifying fractions

Simplifying (or reducing) a fraction means reducing the fraction to an equivalent fraction that uses the lowest possible numbers for the numerator and the denominator.

**This is done by dividing both the numerator and the denominator by the same number.**

We can use a series of small numbers,

numerator

$$\frac{27}{54} \xrightarrow{\div 3} \frac{9}{18} \xrightarrow{\div 3} \frac{3}{6} \xrightarrow{\div 3} \frac{1}{2}$$

denominator

$\div 27$

or, use the largest number possible.

**This largest number is called the highest common factor (HCF) or greatest common factor (GCF).**

$$\frac{15}{20} \xrightarrow{\div 5} = \frac{3}{4} \qquad \frac{16}{40} \xrightarrow{\div 8} = \frac{2}{5} \qquad \frac{25}{100} \xrightarrow{\div 25} = \frac{1}{4}$$

**To find the HCF or GCF ...**

1. List the factors for the numerator and the denominator.

Example   **16**  1, 2, 4, 8, 16

**40**  1, 2, 4, 5, 8, 10, 20, 40

2. List the common factors.

1, 2, 4, (8)

3. Circle the HCF/GCF.

# Adding fractions

## To add fractions ...
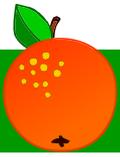
### with the same denominators (like fractions).

- **add the numerators**
- **the denominators stays the same**
- **simplify (reduce) the answer**

**EXAMPLE:** $\dfrac{3}{10} + \dfrac{2}{10} = \dfrac{5}{10} = \dfrac{1}{2}$

### with different denominators (unlike fractions).

- **convert the fractions so they have a common denominator** → **Multiply the denominators**
- **add the numerators**
- **the denominators stay the same**
- **simplify (reduce) the answer**

**EXAMPLE:** $\dfrac{1}{4} + \dfrac{3}{10} = \dfrac{10}{40} + \dfrac{12}{40} = \dfrac{22}{40} = \dfrac{11}{20}$

### To simplify or reduce a fraction.

That is, reduce the numerator and denominator in a fraction to the smallest numbers possible.

- **divide the numerator and denominator by their highest common factor** (HCF or GCF)

$$\dfrac{15 \,\div 5}{20 \,\div 5} = \dfrac{3}{4} \qquad \dfrac{16 \,\div 8}{40 \,\div 8} = \dfrac{2}{5} \qquad \dfrac{25 \,\div 25}{100 \,\div 25} = \dfrac{1}{4}$$

# Subtracting fractions

## To subtract fractions ...

### with the same denominators (like fractions).

- subtract the numerators
- the denominators stays the same
- simplify (reduce) the answer

EXAMPLE: $\dfrac{7}{10} - \dfrac{2}{10} = \dfrac{5}{10} = \dfrac{1}{2}$
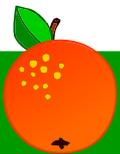
### with different denominators (unlike fractions).

- convert the fractions so they have a common denominator → **Multiply the denominators**
- subtract the numerators
- the denominators stay the same
- simplify (reduce) the answer

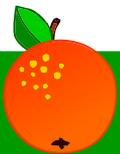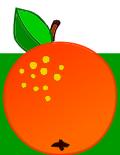EXAMPLE: $\dfrac{3}{4} - \dfrac{1}{10} = \dfrac{30}{40} - \dfrac{4}{40} = \dfrac{26}{40} = \dfrac{13}{20}$

### To simplify or reduce a fraction.

That is, reduce the numerator and denominator in a fraction to the smallest numbers possible.

- divide the numerator and denominator by their highest common factor (HCF or GCF)

$\dfrac{15}{20} \overset{\div 5}{\underset{\div 5}{=}} \dfrac{3}{4}$    $\dfrac{26}{40} \overset{\div 2}{\underset{\div 2}{=}} \dfrac{13}{20}$    $\dfrac{25}{100} \overset{\div 25}{\underset{\div 25}{=}} \dfrac{1}{4}$

# Multiplying fractions

## To multiply a fraction ...

### by a whole number.

- multiply the numerator
- the denominator stays the same
- simplify (reduce) the answer

EXAMPLE:  $5 \times \dfrac{3}{5} = \dfrac{15}{5} = \dfrac{3}{1} = 3$
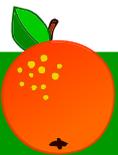
### by another fraction.

- multiply the numerators
- multiply the denominators
- simplify (reduce) the answer

EXAMPLE:  $\dfrac{3}{4} \times \dfrac{4}{5} = \dfrac{12}{20} = \dfrac{3}{5}$

### To simplify or reduce a fraction.

That is, reduce the numerator and denominator in a fraction to the smallest numbers possible.

- divide the numerator and denominator by their highest common factor (HCF or GCF)

$$\dfrac{6}{12} \overset{\div 6}{\underset{\div 6}{=}} \dfrac{1}{2} \qquad \dfrac{15}{20} \overset{\div 5}{\underset{\div 5}{=}} \dfrac{3}{4} \qquad \dfrac{25}{100} \overset{\div 25}{\underset{\div 25}{=}} \dfrac{1}{4}$$

# Dividing fractions

Because division is the opposite of multiplication, to divide a fraction, invert the fraction doing the dividing and then multiply.
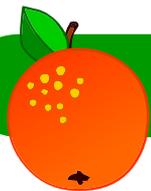
## by another fraction.

- invert the dividing fraction
- multiply the numerators
- multiply the denominators
- simplify (reduce) the answer

EXAMPLE: $\dfrac{3}{4} \div \dfrac{1}{3} = \dfrac{3}{4} \times \dfrac{3}{1} = \dfrac{9}{4}$ OR $2\dfrac{1}{4}$
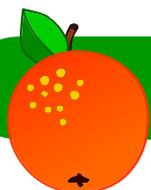
## by a whole number.

- write the whole number as a fraction, invert
- multiply the numerators
- multiply the denominators
- simplify (reduce) the answer if necessary

EXAMPLE: $\dfrac{3}{4} \div 5 = \dfrac{3}{4} \div \dfrac{5}{1} = \dfrac{3}{4} \times \dfrac{1}{5} = \dfrac{3}{20}$

## To simplify or reduce a fraction.

That is, reduce the numerator and denominator in a fraction to the smallest numbers possible.

- divide the numerator and denominator by their highest common factor (HCF or GCF)

$$\dfrac{6 \,\div 6}{12 \,\div 6} = \dfrac{1}{2} \qquad \dfrac{15 \,\div 5}{20 \,\div 5} = \dfrac{3}{4} \qquad \dfrac{25 \,\div 25}{100 \,\div 25} = \dfrac{1}{4}$$

# Fractions of a group

## Unit fractions of a group (whole number).

- **A unit fraction has a numerator of one.**
- **To work out the unit fraction of a whole number divide by the denominator.**

$$\frac{1}{4} \times 20 = 5 \qquad \frac{1}{8} \times 24 = 3 \qquad \frac{1}{10} \times 90 = 9$$

## Proper fractions of a group (whole number).

- **In a proper fraction the numerator is less than the denominator.**
- **To work out the proper fraction of a whole number different methods may be used.**

**METHOD 1:** a. Divide the number by the denominator.
b. Multiply **this answer** by the numerator.

$$\frac{3}{4} \times 20 = 15 \qquad \frac{7}{8} \times 24 = 21 \qquad \frac{9}{10} \times 90 = 81$$

**OR the opposite**  a. Multiply the number by the numerator.
b. Divide the answer by the denominator.

**METHOD 2:** a. Write the whole number as a **fraction.**
b. Multiply the numerators.
c. Multiply the denominators.
d. Simplify (reduce) the answer.

$$\frac{3}{4} \times \frac{20}{1} = \frac{60}{4} = 15 \qquad \frac{9}{10} \times \frac{90}{1} = \frac{810}{10} = 81$$

# Reciprocals of fractions

**Two fractions are reciprocals of each other if their product equals 1.**
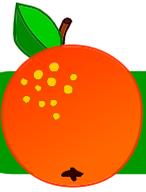
**To get the reciprocal of a fraction ...**

**turn it upside down.**

reciprocal

$$\frac{1}{2} \quad \frac{2}{1} \qquad \frac{1}{2} \times \frac{2}{1} = \frac{2}{2} = 1$$

reciprocal

$$\frac{2}{3} \quad \frac{3}{2} \qquad \frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$$

reciprocal

$$\frac{7}{8} \quad \frac{8}{7} \qquad \frac{7}{8} \times \frac{8}{7} = \frac{56}{56} = 1$$

# Fractions, decimals, percentages

| Fraction | Equivalent Fractions | | | | | Decimal | Percent |
|---|---|---|---|---|---|---|---|
| $\frac{1}{2}$ | $\frac{2}{4}$ | $\frac{3}{6}$ | $\frac{5}{10}$ | $\frac{10}{20}$ | $\frac{50}{100}$ | 0.5 | 50% |
| $\frac{1}{3}$ | $\frac{2}{6}$ | $\frac{3}{9}$ | $\frac{4}{12}$ | $\frac{10}{30}$ | $\frac{30}{90}$ | $0.\overline{3}$ | 33.3% |
| $\frac{2}{3}$ | $\frac{4}{6}$ | $\frac{6}{9}$ | $\frac{8}{12}$ | $\frac{20}{30}$ | $\frac{60}{90}$ | $0.\overline{6}$ | 66.6% |
| $\frac{1}{4}$ | $\frac{2}{8}$ | $\frac{3}{12}$ | $\frac{4}{16}$ | $\frac{10}{40}$ | $\frac{25}{100}$ | 0.25 | 25% |
| $\frac{3}{4}$ | $\frac{6}{8}$ | $\frac{9}{12}$ | $\frac{12}{16}$ | $\frac{30}{40}$ | $\frac{75}{100}$ | 0.75 | 75% |
| $\frac{1}{5}$ | $\frac{2}{10}$ | $\frac{3}{15}$ | $\frac{4}{20}$ | $\frac{10}{50}$ | $\frac{20}{100}$ | 0.2 | 20% |
| $\frac{3}{5}$ | $\frac{6}{10}$ | $\frac{9}{15}$ | $\frac{12}{20}$ | $\frac{30}{50}$ | $\frac{60}{100}$ | 0.6 | 60% |
| $\frac{4}{5}$ | $\frac{8}{10}$ | $\frac{12}{15}$ | $\frac{16}{20}$ | $\frac{40}{50}$ | $\frac{80}{100}$ | 0.8 | 80% |
| $\frac{1}{6}$ | $\frac{2}{12}$ | $\frac{3}{18}$ | $\frac{4}{24}$ | $\frac{10}{60}$ | $\frac{20}{120}$ | $0.1\overline{6}$ | 16.7% |
| $\frac{5}{6}$ | $\frac{10}{12}$ | $\frac{15}{18}$ | $\frac{20}{24}$ | $\frac{50}{60}$ | $\frac{100}{120}$ | $0.8\overline{3}$ | 83.3% |
| $\frac{1}{7}$ | $\frac{2}{14}$ | $\frac{3}{21}$ | $\frac{4}{28}$ | $\frac{10}{70}$ | $\frac{20}{140}$ | $0.\overline{142857}$ | 14.3% |
| $\frac{1}{8}$ | $\frac{2}{16}$ | $\frac{3}{24}$ | $\frac{4}{32}$ | $\frac{10}{80}$ | $\frac{20}{160}$ | 0.125 | 12.5% |
| $\frac{3}{8}$ | $\frac{6}{16}$ | $\frac{9}{24}$ | $\frac{12}{32}$ | $\frac{30}{80}$ | $\frac{60}{160}$ | 0.375 | 37.5% |
| $\frac{5}{8}$ | $\frac{10}{16}$ | $\frac{15}{24}$ | $\frac{20}{32}$ | $\frac{50}{80}$ | $\frac{100}{160}$ | 0.625 | 62.5% |
| $\frac{1}{9}$ | $\frac{2}{18}$ | $\frac{3}{27}$ | $\frac{4}{36}$ | $\frac{10}{90}$ | $\frac{20}{180}$ | $0.\overline{1}$ | 11.1% |
| $\frac{2}{9}$ | $\frac{4}{18}$ | $\frac{6}{27}$ | $\frac{8}{36}$ | $\frac{20}{90}$ | $\frac{40}{180}$ | $0.\overline{2}$ | 22.2% |
| $\frac{4}{9}$ | $\frac{8}{18}$ | $\frac{12}{27}$ | $\frac{16}{36}$ | $\frac{40}{90}$ | $\frac{80}{180}$ | $0.\overline{4}$ | 44.4% |
| $\frac{5}{9}$ | $\frac{10}{18}$ | $\frac{15}{27}$ | $\frac{20}{36}$ | $\frac{50}{90}$ | $\frac{100}{180}$ | $0.\overline{5}$ | 55.5% |
| $\frac{7}{9}$ | $\frac{14}{18}$ | $\frac{21}{27}$ | $\frac{28}{36}$ | $\frac{70}{90}$ | $\frac{140}{180}$ | $0.\overline{7}$ | 77.8% |
| $\frac{8}{9}$ | $\frac{16}{18}$ | $\frac{24}{27}$ | $\frac{32}{36}$ | $\frac{80}{90}$ | $\frac{160}{180}$ | $0.\overline{8}$ | 88.9% |
| $\frac{1}{10}$ | $\frac{2}{20}$ | $\frac{3}{30}$ | $\frac{4}{40}$ | $\frac{5}{50}$ | $\frac{10}{100}$ | 0.1 | 10% |
| $\frac{1}{100}$ | $\frac{2}{200}$ | $\frac{3}{300}$ | $\frac{4}{400}$ | $\frac{5}{500}$ | $\frac{10}{1000}$ | 0.01 | 1% |

A horizontal bar over a digit in a decimal means that digit repeats forever, eg, $0.\overline{3} = 0.333333333 \ldots$
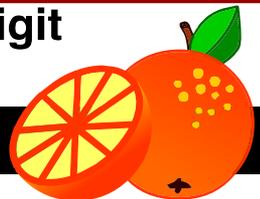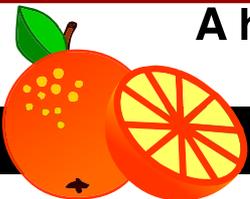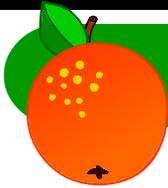
# Fractions, decimals, percentages ... conversions.

## Fraction to Decimal

**Divide the numerator by the denominator.**

$$\frac{1}{4} \qquad 4\overline{)1.00} = 0.25$$

## Decimal to Fraction

**Write the decimal over the number of its place value, then simplify (reduce).**

$$0.25 = \frac{25}{100} = \frac{1}{4}$$
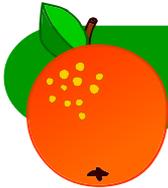
## Fraction to Percent

**Multiply the fraction by 100, simplify (reduce), add the % symbol.**

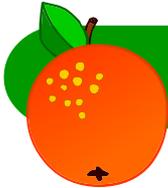$$\frac{1}{4} \times \frac{100}{1} = \frac{100}{4} = 25\%$$

## Percent to Fraction

**Remove the % symbol, write as a fraction with a denominator of 100, then simplify (reduce).**

$$25\% = \frac{25}{100} = \frac{1}{4}$$

**With a decimal percentage, first multiply the numerator and the denominator by 10 until the numerator is a whole number.**

$$2.5\% = \frac{25}{1000} = \frac{1}{40}$$

## Percent to Decimal

**Remove the % symbol, divide the number by 100.**

25%

$$25 \div 100 = 0.25$$

## Decimal to Percent

**Multiply by 100, add the % symbol.**

0.25

$$0.25 \times 100 = 25\%$$