Division 1

6 ÷ 3 = 2

six divided by **three** equals **two**
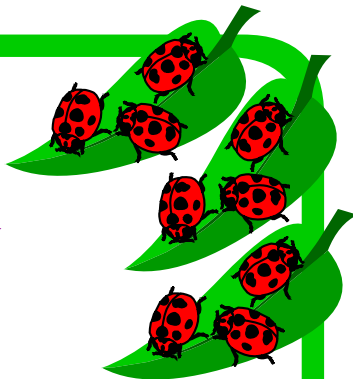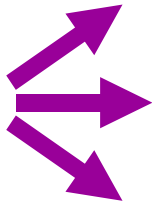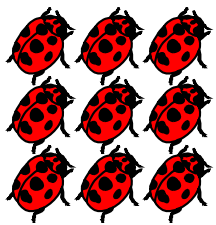
9 ÷ 3 = 3
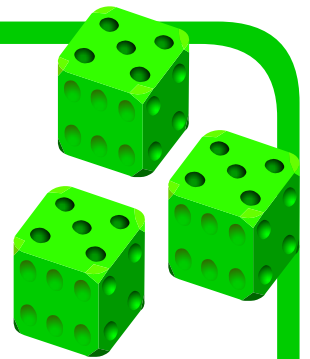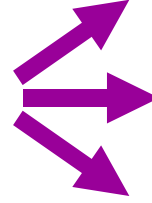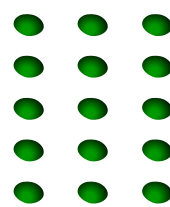
15 ÷ 3 = 5
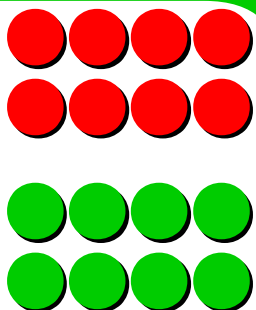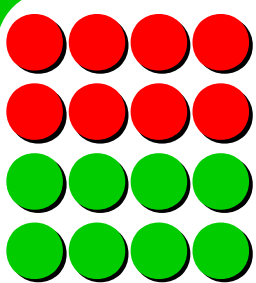
16 ÷ 2 = 8
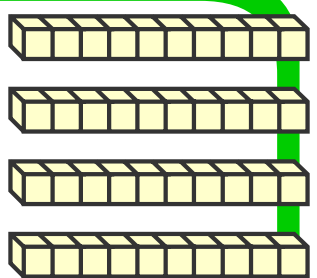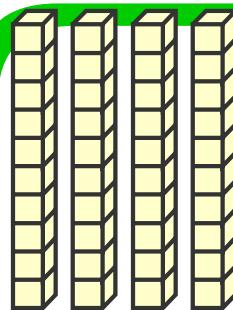
40 ÷ 4 = 10

Division is a mathematical operation which involves sharing or grouping a number into equal parts.

Division 2

$$56 \div 8 = 7$$

dividend   divisor   quotient

Division Key Words: divide, divided by, remainder, dividend, divisor, quotient.

To divide larger numbers a horizontal algorithm is used with a division symbol often called the division bracket.

7   quotient (answer)

divisor **8**) **56** dividend

Start at the left and work to the right.

Any remainders are moved to become the tens in the next place to the right.

Any final remainder may be written as shown.

$$13\,r\,2$$
$$7) 9\overset{2}{3}$$

Remainder as a whole number.

$$239\tfrac{1}{4}$$
$$4) 9\overset{1}{5}\overset{3}{7}$$

Remainder as a fraction.

$$117.25$$
$$8) 9\overset{1}{3}\overset{5}{8}$$

Remainder as a decimal fraction.

Remainders may be written as whole numbers, fractions or decimal fractions.

Using groups and arrays

In early multiplication and division, using groups or arrays of rows and columns make counting and calculating easier.
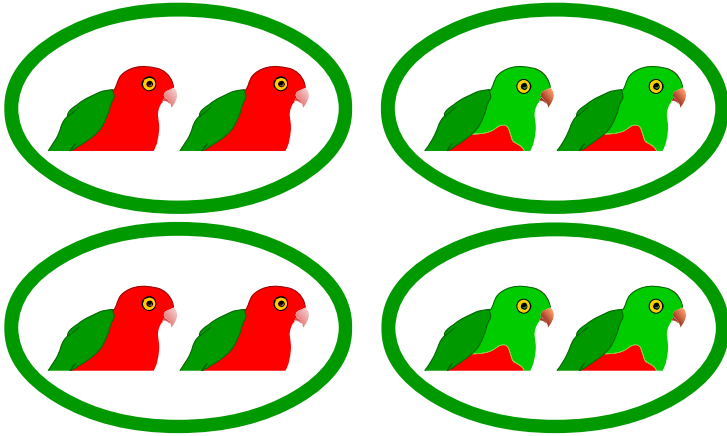
4 groups of 2 = 8

2 x 4 = 8

8 ÷ 4 = 2
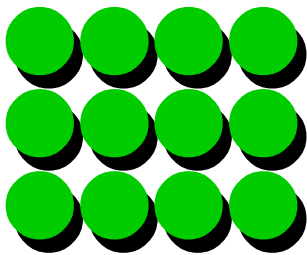
3 rows of 4 = 12

4 x 3 = 12

12 ÷ 3 = 4

3 groups of 3 = 9

3 x 3 = 9
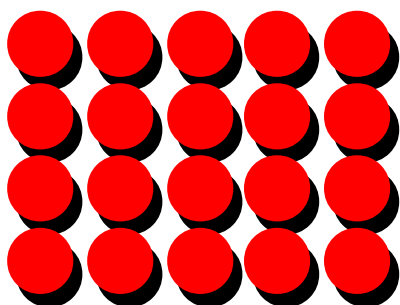
9 ÷ 3 = 3

4 rows of 5 = 20

5 x 4 = 20

20 ÷ 4 = 5

Extensions

A strategy to extend easy number facts to larger numbers using multiplying by 10.

Multiplication
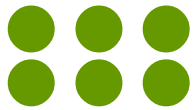
$$2 \times 3 = 6$$
$$2 \times 30 = 60$$
$$2 \times 300 = 600$$
$$2 \times 3000 = 6000$$
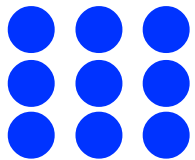
$$3 \times 3 = 9$$
$$3 \times 30 = 90$$
$$3 \times 300 = 900$$
$$3 \times 3000 = 9000$$
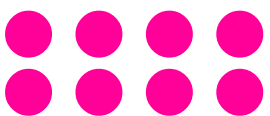
$$2 \times 4 = 8$$
$$2 \times 40 = 80$$
$$2 \times 400 = 800$$
$$2 \times 4000 = 8000$$

Division
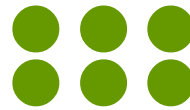
$$6 \div 2 = 3$$
$$60 \div 2 = 30$$
$$600 \div 2 = 300$$
$$6000 \div 2 = 3000$$
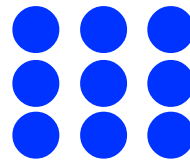
$$9 \div 3 = 3$$
$$90 \div 3 = 30$$
$$900 \div 3 = 300$$
$$9000 \div 3 = 3000$$
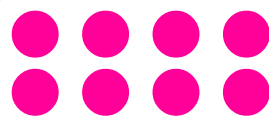
$$8 \div 2 = 4$$
$$80 \div 2 = 40$$
$$800 \div 2 = 400$$
$$8000 \div 2 = 4000$$

Look for the patterns.

Inverse operations

**Inverse operations are opposite or reverse operations.
Multiplication and division are inverse operations.
A multiplication fact will give a division fact and vice versa.**

Multiplication	inverse	Division
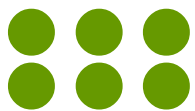
$2 \times 3 = 6$
$3 \times 2 = 6$

$6 \div 2 = 3$
$6 \div 3 = 2$
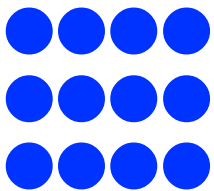
$3 \times 4 = 12$
$4 \times 3 = 12$
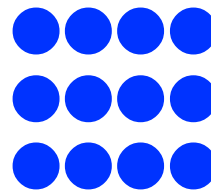
$12 \div 3 = 4$
$12 \div 4 = 3$
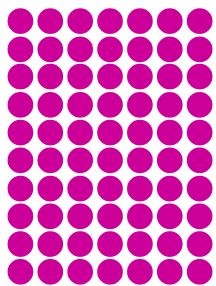
$10 \times 7 = 70$
$7 \times 10 = 70$
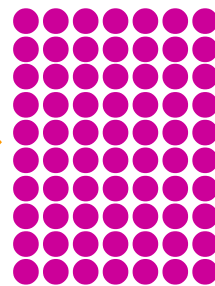
$70 \div 10 = 7$
$70 \div 7 = 10$

$$\begin{array}{r} 213 \\ \times \quad 3 \\ \hline 639 \end{array}$$

$$3\overline{)639} \quad 213$$

**You can use multiplication facts to check division,
or use division facts to check multiplication.**

Split strategy (partitioning)

A way to multiply larger numbers where each digit is multiplied separately according to its place value.

1. Split the larger number into hundreds, tens and ones.
2. Multiply the hundreds, then the tens, then the ones.
3. Add the products together.

EXAMPLES:
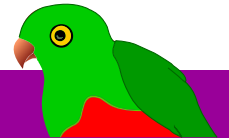
$$165 \times 6 = (100 \times 6) + (60 \times 6) + (5 \times 6)$$
$$= 600 + 360 + 30$$
$$= 990$$

OR

$$165 \times 6 = (100 + 60 + 5) \times 6$$
$$= 600 + 360 + 30$$
$$= 990$$

OR

$$
\begin{array}{r}
165 \\
\times \quad 6 \\
\hline
600 \\
360 \\
30 \\
\hline
990 \\
\hline
\end{array}
$$

Start with the largest place value.

Reordering

A multiplication strategy that changes the order of the numbers to make calculations easier.
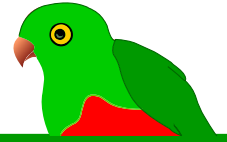
**This strategy is based on the commutative law, rule or property of multiplication.
That is, the product will remain the same regardless of the order of the numbers being multiplied.**

**43 x 3 = 3 x 43
= 129**

**20 x 2 x 4 = 4 x 20 x 2
= 160**

3 x 620 =

$$\begin{array}{r} 620 \\ \times\quad 3 \\ \hline 1860 \end{array}$$

Use whatever order is easier for you.

Area model of multiplication

The area of rectangles is used to model the multiplication of digits in two numbers according to their place value.

It shows the partial products which are then added together to get the answer.

EXAMPLES:

16 x 12

160
+ 32
192

26 x 22

520
+ 52
572
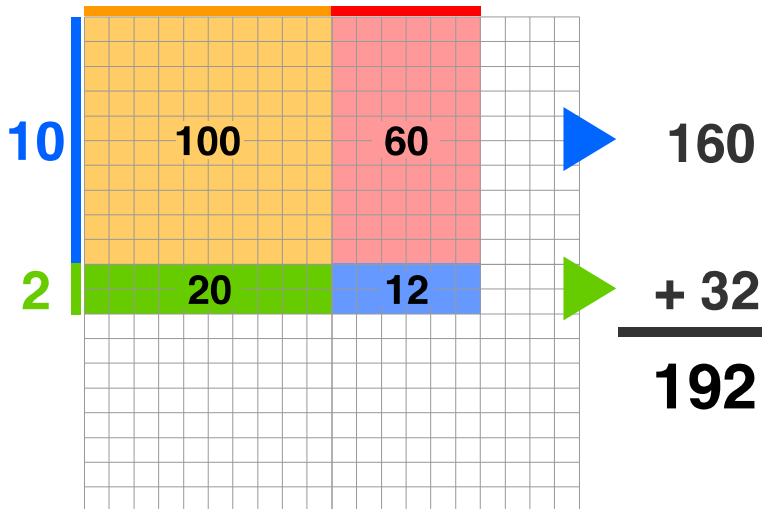
Multiply
26 x 22
(20+6) x (20+2)
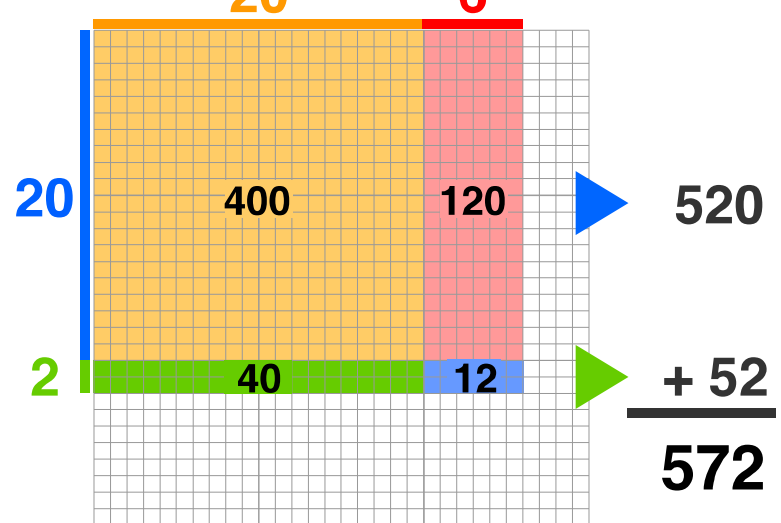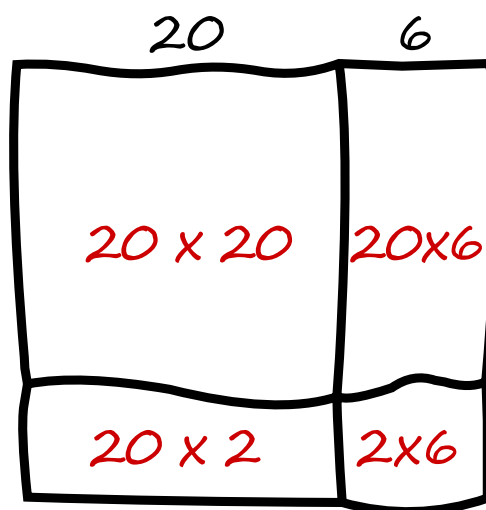
	20	6	*Add*	*Answer*
20	20 x 20	20x6	400 120	
2	20 x 2	2x6	40 12	572

Also called the partial products model.

This model is an application of the distributive property of multiplication which states multiplying a number is the same as multiplying its addends by the number, then adding the partial products.

Multiplication Chart

**Read across and down
to find the product of any two green numbers.**

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

**Read across or down to find the multiples of
any green number.**

Times Tables

1 x 1 = 1	1 x 2 = 2	1 x 3 = 3	1 x 4 = 4
2 x 1 = 2	2 x 2 = 4	2 x 3 = 6	2 x 4 = 8
3 x 1 = 3	3 x 2 = 6	3 x 3 = 9	3 x 4 = 12
4 x 1 = 4	4 x 2 = 8	4 x 3 = 12	4 x 4 = 16
5 x 1 = 5	5 x 2 = 10	5 x 3 = 15	5 x 4 = 20
6 x 1 = 6	6 x 2 = 12	6 x 3 = 18	6 x 4 = 24
7 x 1 = 7	7 x 2 = 14	7 x 3 = 21	7 x 4 = 28
8 x 1 = 8	8 x 2 = 16	8 x 3 = 24	8 x 4 = 32
9 x 1 = 9	9 x 2 = 18	9 x 3 = 27	9 x 4 = 36
10 x 1 = 10	10 x 2 = 20	10 x 3 = 30	10 x 4 = 40
11 x 1 = 11	11 x 2 = 22	11 x 3 = 33	11 x 4 = 44
12 x 1 = 12	12 x 2 = 24	12 x 3 = 36	12 x 4 = 48
1 x 5 = 5	1 x 6 = 6	1 x 7 = 7	1 x 8 = 8
2 x 5 = 10	2 x 6 = 12	2 x 7 = 14	2 x 8 = 16
3 x 5 = 15	3 x 6 = 18	3 x 7 = 21	3 x 8 = 24
4 x 5 = 20	4 x 6 = 24	4 x 7 = 28	4 x 8 = 32
5 x 5 = 25	5 x 6 = 30	5 x 7 = 35	5 x 8 = 40
6 x 5 = 30	6 x 6 = 36	6 x 7 = 42	6 x 8 = 48
7 x 5 = 35	7 x 6 = 42	7 x 7 = 49	7 x 8 = 56
8 x 5 = 40	8 x 6 = 48	8 x 7 = 56	8 x 8 = 64
9 x 5 = 45	9 x 6 = 54	9 x 7 = 63	9 x 8 = 72
10 x 5 = 50	10 x 6 = 60	10 x 7 = 70	10 x 8 = 80
11 x 5 = 55	11 x 6 = 66	11 x 7 = 77	11 x 8 = 88
12 x 5 = 60	12 x 6 = 72	12 x 7 = 84	12 x 8 = 96
1 x 9 = 9	1 x 10 = 10	1 x 11 = 11	1 x 12 = 12
2 x 9 = 18	2 x 10 = 20	2 x 11 = 22	2 x 12 = 24
3 x 9 = 27	3 x 10 = 30	3 x 11 = 33	3 x 12 = 36
4 x 9 = 36	4 x 10 = 40	4 x 11 = 44	4 x 12 = 48
5 x 9 = 45	5 x 10 = 50	5 x 11 = 55	5 x 12 = 60
6 x 9 = 54	6 x 10 = 60	6 x 11 = 66	6 x 12 = 72
7 x 9 = 63	7 x 10 = 70	7 x 11 = 77	7 x 12 = 84
8 x 9 = 72	8 x 10 = 80	8 x 11 = 88	8 x 12 = 96
9 x 9 = 81	9 x 10 = 90	9 x 11 = 99	9 x 12 = 108
10 x 9 = 90	10 x 10 = 100	10 x 11 = 110	10 x 12 = 120
11 x 9 = 99	11 x 10 = 110	11 x 11 = 121	11 x 12 = 132
12 x 9 = 108	12 x 10 = 120	12 x 11 = 132	12 x 12 = 144

Multiples and LCM

The multiple of a number is the product of multiplying that number by another whole number.

Multiples Chart

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

The LCM is the least or lowest number that is a common multiple of two or more numbers.

Finding the LCM

1. List the multiples for each number.

   4   4, 8, 12, 16, 20, 24, 28, 32, 36, ...

   6   6, 12, 18, 24, 30, 36, 42, 48, ...

2. List the common multiples in order.

   (12,) 24, 36, …

3. Record the lowest.

   LCM = 12

Factors and HCF

A factor is a whole number that divides exactly into another given number.

That is, a whole number that multiplies with another number to make a third number.

To find the factors of a number, first divide by 1 and then keep working down using the next lowest number that will divide exactly. Record each factor pair.

20

1 x 20
2 x 10
4 x 5

24

1 x 24
2 x 12
3 x 8
4 x 6

48

1 x 48
2 x 24
3 x 16
4 x 12
6 x 8

96

1 x 96
2 x 48
3 x 32
4 x 24
6 x 16
8 x 12

The HCF or GCF is the highest or greatest common factor that will divide two or more other numbers exactly.

Finding the HCF or GCF

1. List the factors for each number.

20 1, 2, 4, 5, 10, 20
24 1, 2, 3, 4, 6, 8, 12, 24

2. List the common factors.

1, 2, ④

3. Record the highest.

HCF or GCF = 4

Prime factors and HCF

A prime factor is a prime number that divides exactly into another given number. Every positive integer has its own unique set of prime factors.

In prime factorisation, a number is written as the product of its prime factors.
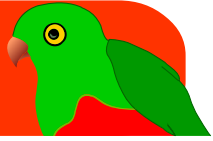
To find the prime factors of a composite number, first divide the number by 2 and then keep working down using 2 or the next lowest prime number that will divide exactly, until there are no composite factors left.

20
2 x 10
2 x 5
20=2x2x5

24
2 x 12
2 x 6
2 x 3
24=2x2x2x3

48
2 x 24
2 x 12
2 x 6
2 x 3
48=2x2x2x2x3

75
3 x 25
5 x 5
75=3x5x5

The HCF (highest common factor) or GCF (greatest common factor) is the product of all the prime factors two or more numbers have in common.

Finding the HCF or GCF

1. List the prime factors for each number.

20 2 x 2 x 5
48 2 x 2 x 2 x 2 x 3

2. Find the common prime factors.

⬇ ⬇
2 x 2

3. Calculate their product. **HCF or GCF = 4**

Divisibility rules

A number is divisible by another number if it can be divided exactly without leaving a remainder.

Divisibility Chart

A number is divisible by ...	If ...	Divisible	Not Divisible
2	the last digit is divisible by 2	8724	8721
3	the **sum** of the digits is divisible by 3	8724 21	8722
4	the number made by the last two digits is divisible by 4	8724	8723
5	the last digit is 0 or 5	8725	8724
6	the number is divisible by both 2 and 3	8724	8722
7	the number is 0 or divisible by 7, after removing, doubling and subtracting the last digit from the number	8722 868	8724
8	the number made by the last three digits is divisible by 8	8720	8724
9	the **sum** of the digits is divisible by 9	8721 18	8724
10	the last digit is 0	8720	8724

Long multiplication

A traditional method used to multiply by a number which contains more than one digit.

The larger number to be multiplied is usually written on the top line with the smaller number written underneath.

Numbers are written underneath each other according their place value. The numbers are multiplied vertically, starting with the ones column then moving left column by column.

The multiplication lines are then added together to give the final answer.
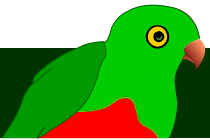
Example

$43,864 x 423 =

Multiply by the digit in:

- **the 1s column**
- **the 10s column**
- **the 100s column**

```
      43864
  x     423
      ¹²¹¹
    131592
     ¹¹
    877280
   ¹³²¹
  17545600
   ¹¹¹¹
 18554472
```

Add the results

Zero or blank spaces may be used as placeholders.

$43,864 x 423 = $18,554,472

When a column is more than ten, the tens go into the next column left and are added to the answer in that line, the ones stay in their own column.

Long division
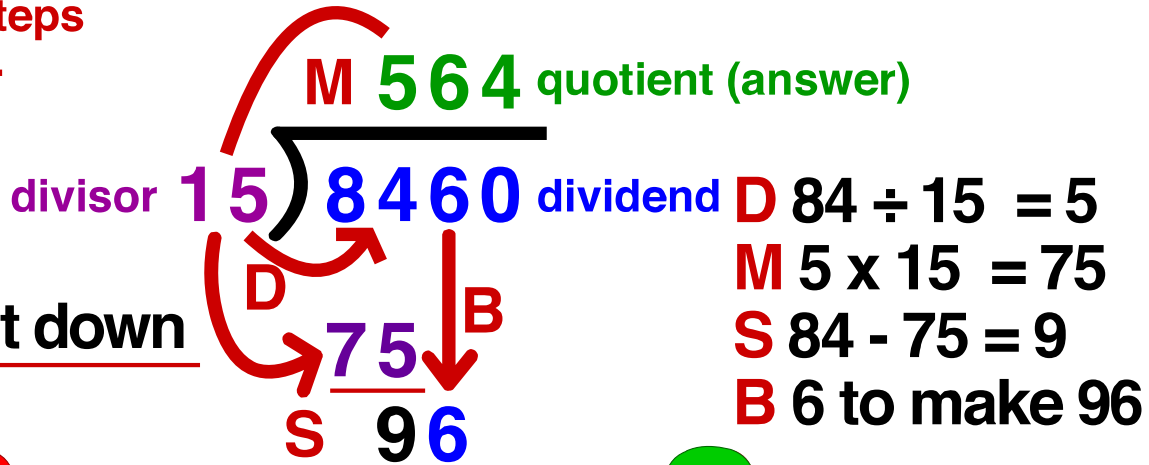
To divide larger numbers a horizontal algorithm is used with a division symbol often called the division bracket.

Long Division Steps

Repeat the first four steps as often as necessary.

1. **D** ivide
2. **M** ultiply
3. **S** ubtract
4. **B** ring next digit down
   R emainder

M **564** quotient (answer)

divisor **15**) **8460** dividend

D **75**

S **96**

D 84 ÷ 15 = 5
M 5 x 15 = 75
S 84 - 75 = 9
B 6 to make 96

Example

$$1225\frac{13}{21}$$

21) 25738
  -21
    47
   -42
     53
    -42
     118
    -105
      13

To calculate the final remainder as a decimal fraction, add a decimal point to the dividend and the quotient. Then add zeros as necessary and keep dividing until the required number of decimal places is reached.

1225.619
21) 25738.0000

1225.62
21) 25738.000

The final remainder may be written as a fraction or decimal fraction, whole number(s) after the letter R or may be used to round the quotient (answer) to a whole number.

Key Words

Key words can help us solve mathematical word problems.

Addition

add	combined	more, more than
add to, added to	extra	plus
addition of	in all	sum
altogether	increase by	together
and	increased by	total, total of
both	join	whole amount
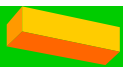
Subtraction

decrease, decreased	how much less	minus
deduct	how many more	nearer, further
difference	how much more	reduce, reduced by
difference between	how many left	remaining
dropped, fell	how much left	subtract
fewer, fewer than	less, less than	take away

Multiplication

area of	multiplied by	rows of
at	of	sets of
by	multiple	times
double, doubled	per	triple
each had	product, product of	twice
groups of	rate	

Division

divide evenly	for each, per	quotient
divided by	half, quarter	ratio, ratio of
equal parts	how many each	share, share of
equal pieces	out of	shared equally
fraction	percent, percentage	split

Equals

answer to	is equal to	result is, results in
corresponds to	is identical to	same amount, value
equals	is, are, would be	the same, same as
equates to	makes	yields, produces
gives	matches	

Problem Solving

Following steps and using strategies can help us solve mathematical problems, particularly unfamiliar ones.

for word problems

the question

- **C**ircle the numbers
- **U**nderline key words
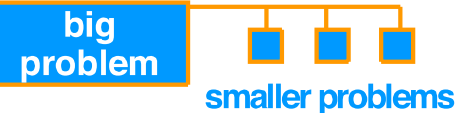- **B**ox the question
- **E**liminate irrelevant info

Q

Lily had ⑤ cookies.
She gave some away to Liam.
She had ① cookie left.
How many cookies did Liam get?

the strategies

- **A**pply known methods, **or**
- **R**esearch strategies
- **C**hoose a strategy

S

③ ❓ ⑨ ⑫

find a pattern or rule

draw a picture

x^2

2 → 4
3 → ?
? → ?

draw a diagram

work backwards

$$5 - \boxed{?} = 1$$

write an equation

$$198 - 72 = ?$$
$$200 - 70 = 130$$
estimated
$$198 - 72 = 126$$
checked

guess and check

act it out

? ➕ ➖ ✖️ ➗

which operation(s)?

big problem → smaller problems

break it down

Animal	Sightings
crabs	5
dolphins	4
sharks	6

make a list or table

25% 20% 15% 10% 30%
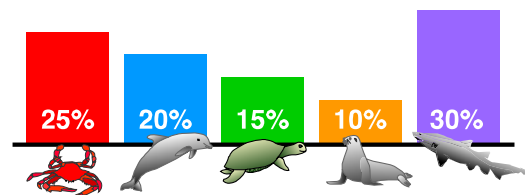
draw a graph or chart

the answer

- Write your answer
- Check the maths ✓
- Does it make sense? ✓

A

Liam got 4 cookies. ✓

- Explain your reasoning.

Fact families

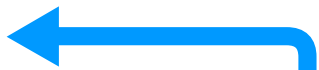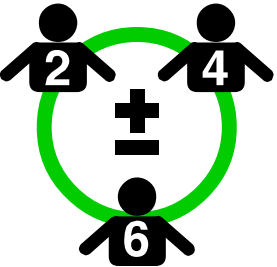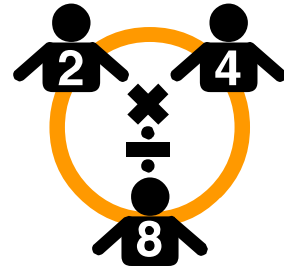**A fact family is made up of four facts related by addition and subtraction or multiplication and division. Each fact family consists of a pair of numbers plus the answer when they are either added or multiplied.**

For example, 2 and 4 with 6 if they are added or 2 and 4 with 8 if they are multiplied.

Knowing one fact can help work out the other facts.

EXAMPLES: Addition and subtraction

| **2, 4, and 6** | 2 + 4 = 6 | 4 + 2 = 6 | 6 - 2 = 4 | 6 - 4 = 2 |
| **3, 5, and 8** | 3 + 5 = 8 | 5 + 3 = 8 | 8 - 3 = 5 | 8 - 5 = 3 |

EXAMPLES: Multiplication and division

| **2, 4, and 8** | 2 x 4 = 8 | 4 x 2 = 8 | 8 ÷ 2 = 4 | 8 ÷ 4 = 2 |
| **3, 5, and 15** | 3 x 5 = 15 | 5 x 3 = 15 | 15 ÷ 3 = 5 | 15 ÷ 5 = 3 |

Fact family relationships

9 **10**
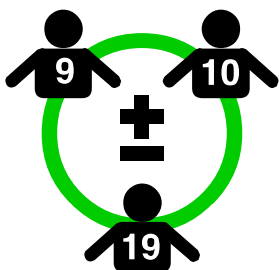
FACT FAMILY: Addition and subtraction

9 + 10 = 19   10 + 9 = 19

19 - 9 = 10   19 - 10 = 9

FACT FAMILY: Multiplication and division

9 x 10 = 90   10 x 9 = 90

90 ÷ 9 = 10   90 ÷ 10 = 9
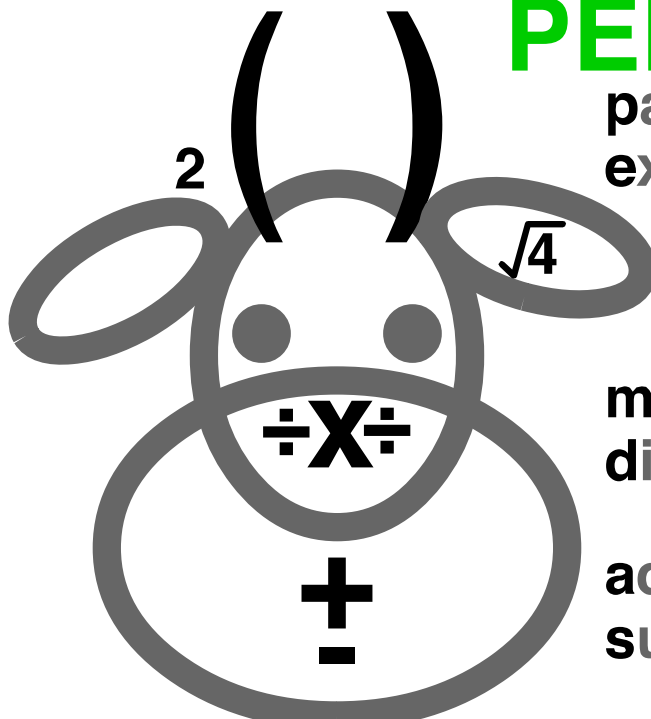
Order of operations

Mathematical operations need to be done in the right order.

Often acronyms such as PEMDAS, BIDMAS or BODMAS are used to help remember the sequence.

PEMDAS

parentheses
exponents, roots

multiplication
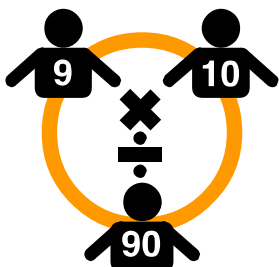**d**ivision

addition
subtraction

e.g. $2 + 6 (3+1)^2$

P = $2 + 6 (4)^2$

E = $2 + 6 (16)$

M = $2 + 96$

D

A = 98

S

wrong order ... wrong answer

PEMDAS

1. **Parentheses ( ) or {} or [ ] , brackets**
2. **Exponents (indices, orders), roots**
3. **Multiplication (times)**   x and ÷ have
   Division (divided by)   equal precedence
4. **Addition (plus)**   + and - have
   Subtraction (minus)   equal precedence

Use the PEMDAS order.

Operations properties

Addition

Associative property

$(a + b) + c = a + (b + c)$ $(4 + 2) + 1 = 4 + (2 + 1)$

Commutative property

$a + b = b + a$ $4 + 2 = 2 + 4 = 6$

Additive identity property of 0

$a + 0 = 0 + a = a$ $4 + 0 = 0 + 4 = 4$

Multiplication

Associative property

$(a × b) × c = a × (b × c)$ $(4 × 2) × 1 = 4 × (2 × 1)$

Commutative property

$a × b = b × a$ $4 × 2 = 2 × 4 = 8$

Multiplicative identity property of 1

$a × 1 = 1 × a = a$ $4 × 1 = 1 × 4 = 4$

Zero product property

$a × b = 0$ either $a = 0$, $b = 0$ or both a and $b = 0$

Distributive property of multiplication over addition

$a × (b + c) = a × b + a × c$ $4 × (2 + 1) = 4 × 2 + 4 × 1$

Inverses

Additive inverses

$a + (–a) = (–a) + a = 0$ $4 + (–4) = (–4) + 4 = 0$

Multiplicative inverses

$a × 1/a = 1/a × a = 1$ $4 × 1/4 = 1/4 × 4 = 1$
if $a ≠ 0$

Operations on positive and negative numbers

Addition
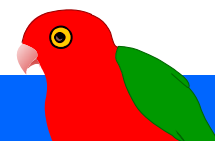
Positive + Positive = Positive

Negative + Negative = Negative

* Positive + Negative or Negative + Positive

• subtract the smaller number from the larger number,

• then use the sign of the larger number in the answer

$5 + 3 = 8$

$(- 5) + (- 3) = - 8$

$(- 5) + 3 = - 2$

$3 + (- 5) = - 2$

$(- 3) + 5 = 2$

$5 + (- 3) = 2$

Subtraction

Negative - Positive = Negative

Positive - Negative = Positive

* Negative - Negative = Negative + Positive

• treat as Negative + Positive

• subtract the smaller number from the larger number,

• then use the sign of the larger number in the answer

$(- 5) - 3 = (- 5) + (- 3) = - 8$

$5 - (- 3) = 5 + 3 = 8$

$(- 5) - (- 3) = (- 5) + 3 = - 2$

$(- 3) - (- 5) = (- 3) + 5 = 2$

Multiplication

Positive x Positive = Positive

Negative x Negative = Positive

Negative x Positive = Negative

Positive x Negative = Negative

• change double negatives to a positive

$5 \times 3 = 15$

$(- 3) \times (- 5) = 15$

$(- 3) \times 5 = - 15$

$3 \times (- 5) = - 15$

Division

Positive ÷ Positive = Positive

Negative ÷ Negative = Positive

Negative ÷ Positive = Negative

Positive ÷ Negative = Negative

• change double negatives to a positive

$15 ÷ 3 = 5$

$(- 15) ÷ (- 3) = 5$

$(- 15) ÷ 3 = - 5$

$15 ÷ (- 3) = - 5$